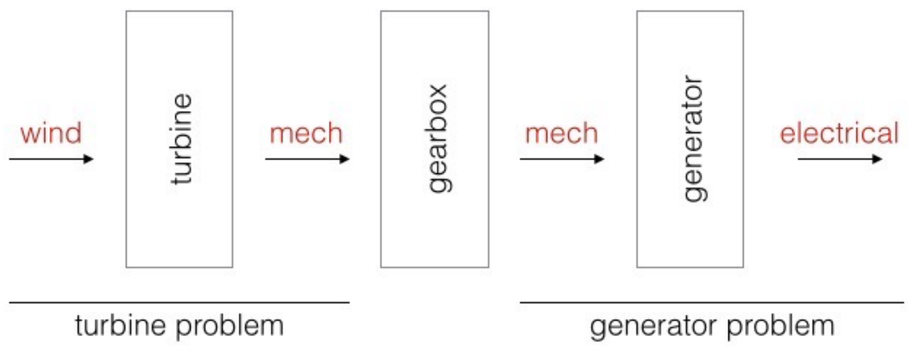
MEAM 347
VAWT Final Report
Tighe Costa, Sean Kelly

Introduction

Student teams design and build a vertical axis wind turbine and transmission to generate electrical power. Each team tests its turbine on a human-powered indoor mobile base. Teams predict turbine performance prior to full-scale testing based on wind-tunnel experimentation and analysis. Each team also selects a load resistor to which the electrical power generated will be delivered. The goal is to maximize electrical power generation.

Background Information

The VAWT problem can be divided into two discrete parts: the "turbine problem" (design a turbine for maximum mechanical power output), and the "generator problem" (given a mechanical power input, find the external resistance that yields maximum electrical power output). The power output of the turbine is connected to the power input of the generator through the gearbox.



The Turbine Problem

Vertical axis wind turbines fall into two categories: lift based (Darrieus) VAWTs and drag based (Savonius) VAWTs. Darrieus VAWTs rely on the lift generated by the turbine blades to rotate. Darrieus turbines are not self-starting and have maximum efficiency at high speed. Savonius VAWTs rely on a differential drag between the bluff and streamlined sides of the turbine blades to rotate. Savonius turbines are self-starting and have maximum efficiency at low speed.[1]

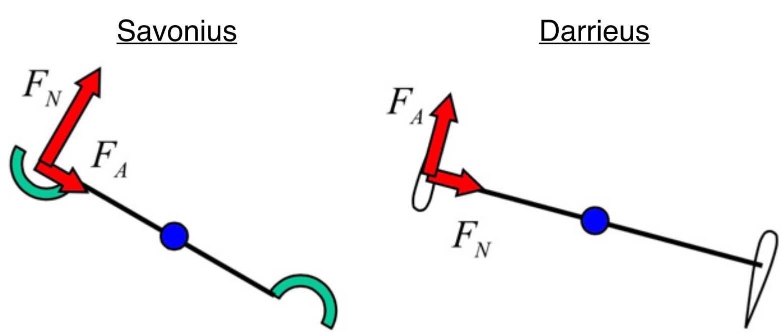


FIGURE 1

VAWTs with vertical blades experience cyclical power output, as the torque on the turbine varies with the angle of the turbine. Commercial VAWTs typically use helical blades to smooth the torque variation, however helical blades are difficult to design and manufacture and the team decided not to pursue helical geometry due to time constraints.

Letcher investigates the design of a turbine for low wind speeds (8-12mph) for maximum efficiency and manufacturability. His final design is a hybrid VAWT, a two-stage Savonius turbine with offset rotors with a Darrieus turbine on the same axis.[2] Research on the design and optimization of Darrieus turbines, especially *Aerodynamic Shape Optimization of a Vertical-Axis Wind Turbine Using Differential Evolution*,[3] led the team to decide against building a Darrieus turbine due to time constraints and low manufacturability.

Savonius rotors be improved by changing blade profile from the traditional circular arc to a combination arc and straight line (A.I Fig. 1),[4] adding converging nozzles (A.I Fig. 2) to increase the velocity of the air as it flows through the torque side,[5] and a drag shield (A.I Fig. 3) to deflect air from the anti-torque side of the turbine.[6] Time constraints led the team to abandon its initial plan to build a two-stage modified-blade-profile Savonius turbine with a combination converging nozzle/drag shield (A.I Fig. 4) and instead focus on the geometric optimization of a single-stage, circular profile Savonius rotor.

The Generator Problem

The generator used to measure the full-scale turbine's performance is a brushed DC motor run in reverse. Rotating the shaft of a brushed DC motor induces a potential difference across the motor's brushes. Brushed DC motors can be modeled as a voltage source in series with an inductor and a resistor (Fig. 2). The equations governing brushed DC motor performance are

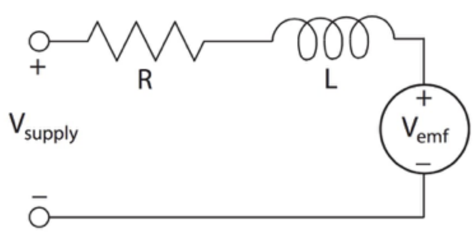


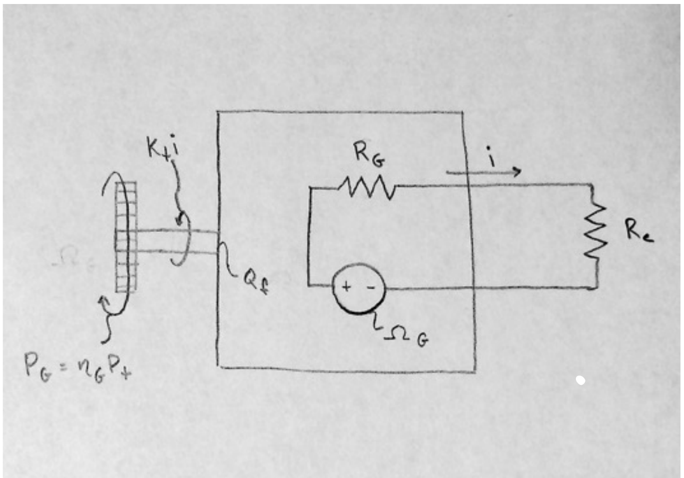
FIGURE 2

$$Q = K_t i$$
$$V_{\text{emf}} = K_e \Omega$$
$$V_{\text{supply}} = Ri + L\frac{di}{dt} + V_{\text{emf}}$$

The team assumed a steady-state condition in evaluating the generator problem since the time scale of the VAWT project is large enough that transient inductive effects of the generator are negligible and can be ignored.[7] The steady-state assumption means that the performance characterizing parameters of a brushed DC motor are friction torque (Q_f), internal resistance (R_i), torque constant (K_t), and speed constant (K_e).

The steady-state generator can be accurately modeled as a voltage source in series with two resistors (Fig. 3), where electrical power output is measured across the external resistor (R_e). A system of four equations and five unknowns, derived from the governing equations above and Ohm's law, allows for optimization of R_e for a given mechanical input power to maximize electrical output power across the external resistor.

Since the power-torque curve of a brushed DC generator is an inverted parabola, for every input power there is an optimal external resistance that governs the current through the generator so that it is operating at the torque corresponding to the most power. The team wrote a MATLAB program that takes estimated turbine power (P_t) as input and outputs an optimized generator torque (Q_G), spindle speed (Ω_G), current (i), and external resistance (R_e). See Appendix III for code.

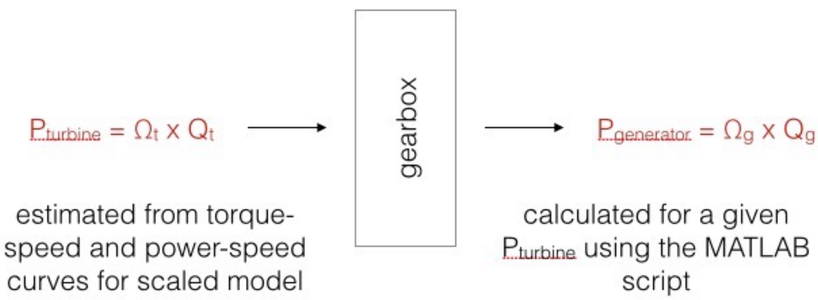


$$Q_G \Omega_G = i^2 (R_G + R_e)$$
$$P_t = Q_f \Omega_G + i^2 (R_G + R_e)$$
$$k_t i = Q_G - Q_f$$
$$P_{out} = i^2 R_e$$

FIGURE 3

Transmission

The purpose of the transmission is to allow both the turbine and the generator to operate at their maximum power torque. The transmission ratio is simply the ratio of the turbine output torque to the generator input torque. The MATLAB script that solves the generator problem also takes the estimated turbine torque (Q_t) and outputs the transmission ratio.



gearbox ratio:

$$G = \frac{Q_t}{Q_g}$$

Experimental Methods

Experiments were centered around the geometric optimization of a single-stage, circular blade profile Savonius rotor. To understand how different geometric parameters effect turbine performance, the team constructed small-scale turbines from MDF and balsa wood and experimentally determined each turbine's power-speed curve using the wind tunnel in B2.

The turbines were mounted to a shaft connected to a brushed DC motor outside the test area. An ammeter was connected to measure current through the motor, and a laser tachometer was used to measure the speed of the turbine.

Varying the external resistance of the motor allowed the team to get torque-speed data in the middle range for each turbine. To get characterization data at torques below the friction torque of the wind tunnel motor, the team put a power supply across the leads of the motor to apply a variable torque in the direction of motion of the turbine and cancel out the friction torque. To get data at torques higher than possible by shorting the leads of the motor together, the team put a power supply across the leads of the motor to apply a variable torque against the direction of motion of the turbine and supply additional torque. The combination of these three methods allowed the team to get data across the entire torque-speed curve for each turbine.

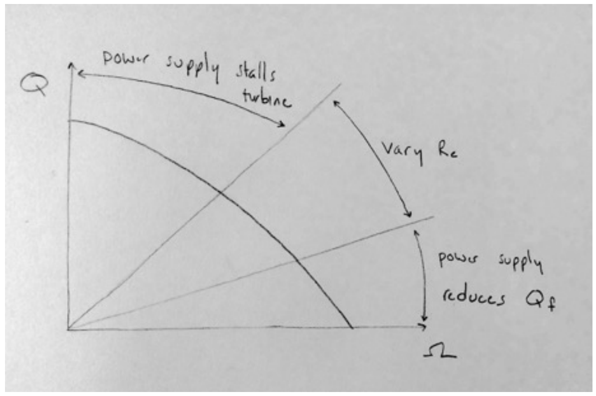


FIGURE 4

Turbines were modeled parametrically in SolidWorks. Turbine geometry was defined by three parameters: turbine diameter, blade depth, and height (Fig. 5). The chord of each blade was set to be 62.5% of the turbine diameter, so that the blade overlap would be 25% of the turbine diameter. The blade depth was the ratio of the depth of each blade to the diameter of that blade. The team made eight turbines all together, uniquely varying each parameter across at least three turbines.

FIGURE 5

The motor in the wind tunnel was characterized as follows:

- R_i — measured resistance across leads of motor in two different shaft positions (~90° apart) and averaged measurements
- K_e — applied different voltages to motor, solved for V_{emf} by subtracting no-spin voltage, then divided by measured spindle speed and averaged five calculations
- Q_f — multiplied K_t (found from K_e) by no-spin current

The test-day characterization was a combination of data from Kevin Muller (K_e, K_t), Pawel Saffron (R_i), and a no-spin current measurement.

Results
The data acquired from the wind tunnel experiments was nondimensionalized using the following equations:

$$\lambda = \frac{\Omega r}{V} \qquad C_Q = \frac{Q}{\frac{1}{2}\rho V^2 A r} \qquad C_P = \frac{P}{\frac{1}{2}\rho V^3 A}$$

By normalizing the characterizing curves of the turbines, the differences in area and radius of the turbines tested are compensated for and the turbine scale is removed from the comparison.

Diameter Variation

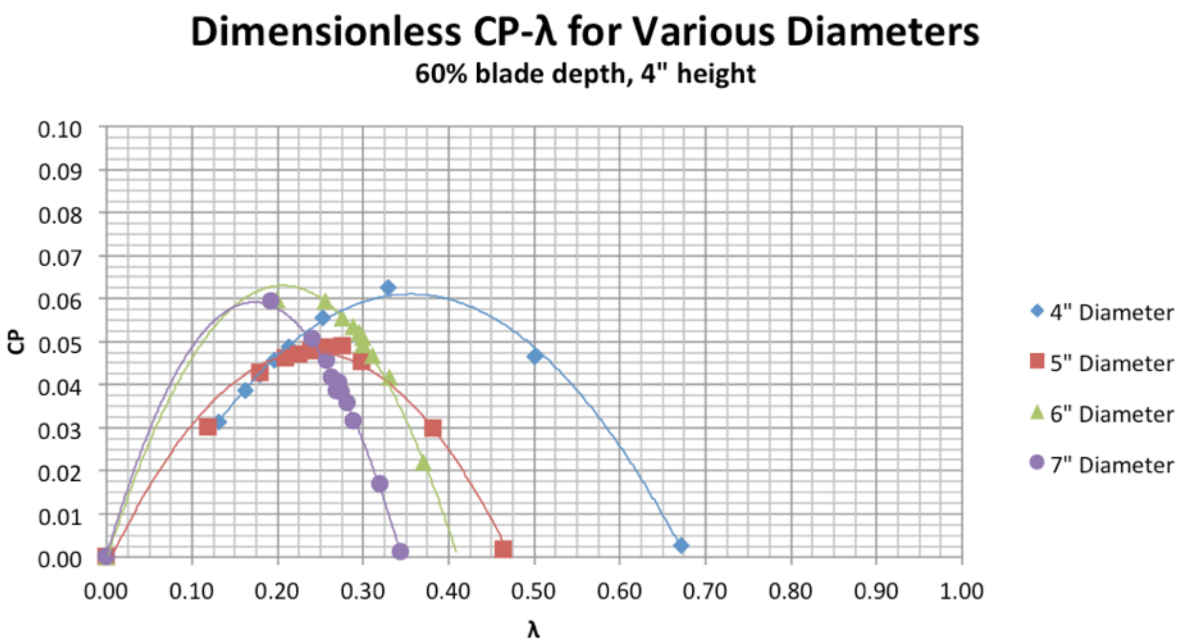


FIGURE 6

Increasing the diameter of the turbine does not change the maximum coefficient of power, but does increase the tip speed ratio at which that power occurs. The 5" diameter turbine has the lowest coefficient of power, however this seems to be an anomaly as the other turbines do not follow this trend.

Height Variation

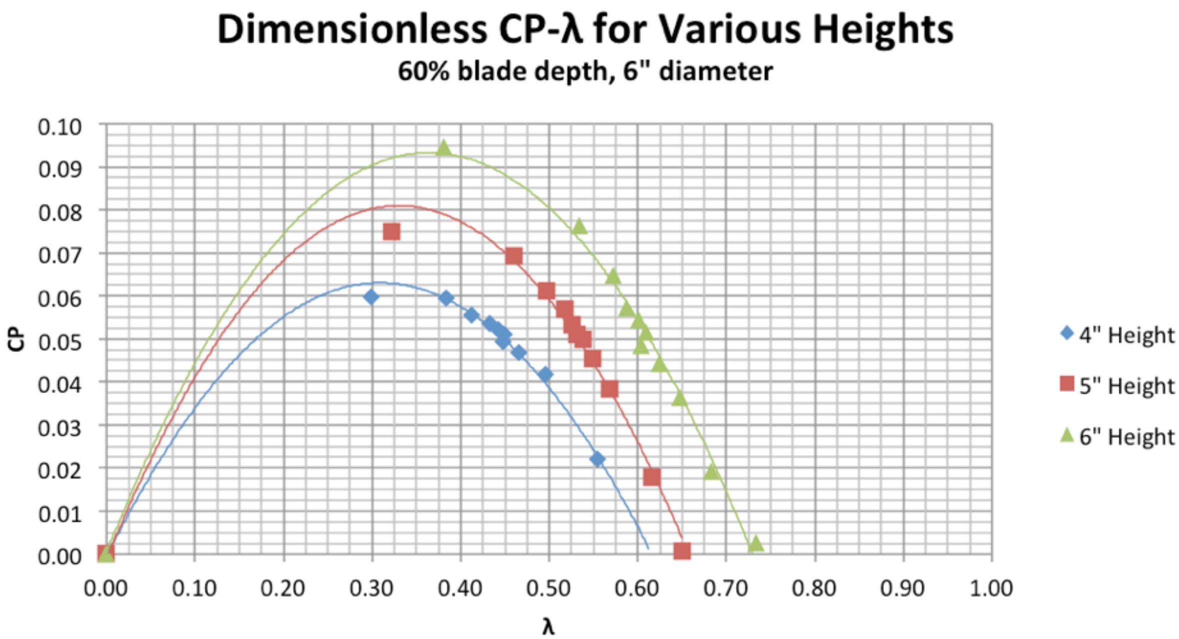


FIGURE 7

Increasing the height of the turbine increases the maximum coefficient of power and the corresponding tip speed ratio.

Cup Depth Variation

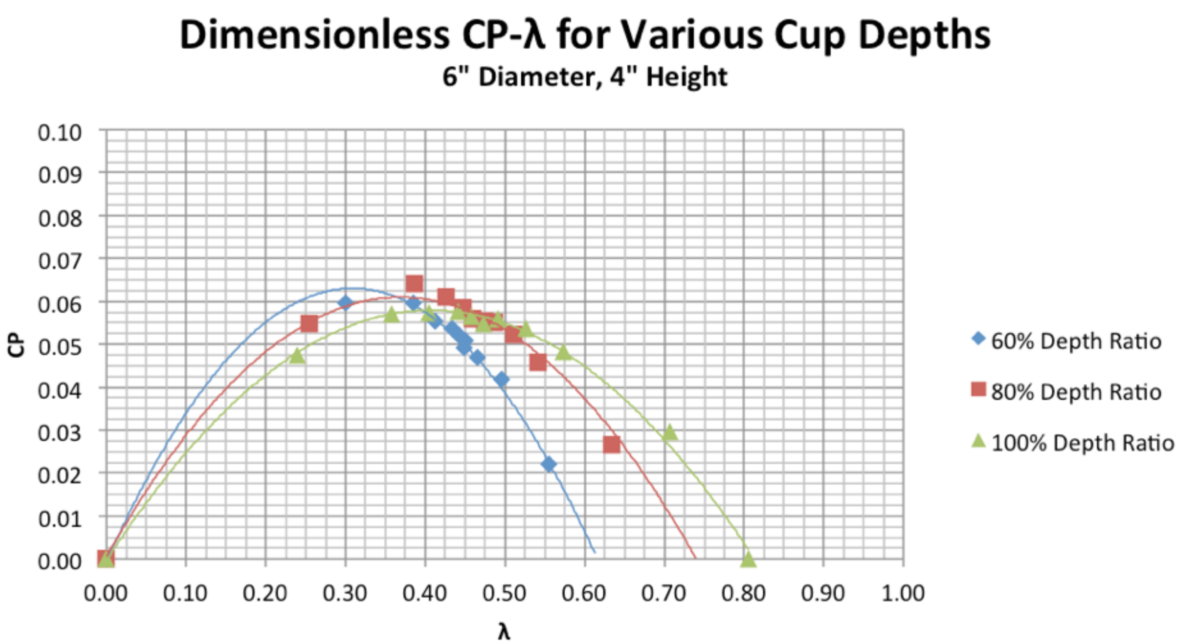


FIGURE 8

Increasing the cup depth ratio of the blades decreased the maximum coefficient of power, and increased the tip speed ratio at which that power occurs.

Analysis

The wind tunnel testing led the team to the following three conclusions about Savonius wind turbine geometry:

- the overall diameter of the turbine does not effect C_P, just the λ at which C_P is at a maximum
- increasing the height of the Savonius turbine increases C_P and λ
- decreasing the cup depth ratio of the blades increases C_P and decreases λ at which C_P is a maximum

Increasing the diameter of the turbine increases the drag force differential, but also increases the distance from the center of rotation of the line of action of the drag force. Since turbine power is the product of turbine torque and speed, the increased torque due should be balanced out by the decreased speed, so C_P would remain the same. The decreased speed leads to a lower corresponding λ, since larger-diameter turbines produce more power at high torque-low speed. The 5" diameter outlier suggests an experimental error, which the team suspects may have to do with a mistake in wind speed adjustment (which had to be reset for each turbine).

While taller turbines produce more power, the team initially expected that the theoretical no-slip boundary conditions of the wind tunnel would mean that taller turbines have a lower C_P since the velocity of the air is greatest in the center of the tunnel and increasing the height would decrease the overall average wind speed the turbine experiences and C_P normalizes out the size of the turbine. However, taller turbines experience greater drag differential without increasing the distance from the center of rotation of the line of action of the drag force, so have both increased torque and increased speed. This drastically outweighed any boundary effects. The increase in λ with turbine height was expected, for similar reasons.

The team regrets not having made turbines with lower cup depth ratios to see if a 60% ratio yielded the maximum C_P or if the trend would have continued through ratios below 60%. Given that a Savonius turbine is more efficient with combination arc and straight line blade profiles than circular profiles,[8] the team suspects that the higher efficiency of the lower cup depth ratios is due to the way the blades direct air flow between them.

The team combined these three conclusions to design the full-scale turbine: maximize turbine height, then maximize turbine diameter, and use a cup depth ratio of 60%. The height (48") was determined by the length of the steel axle, the diameter (58") was determined from the distance from the axle mount to the handlebars of the human-powered cart.

The team used the dimensionless data from the 4"x4", 60% cup depth ratio turbine to predict the performance of the full-scale turbine, since it was closest to the aspect ratio of the final turbine and the team did not have time to test another small-scale turbine. The maximum C_P is 0.061, corresponding to a λ of 0.355 and a C_Q of 0.175.

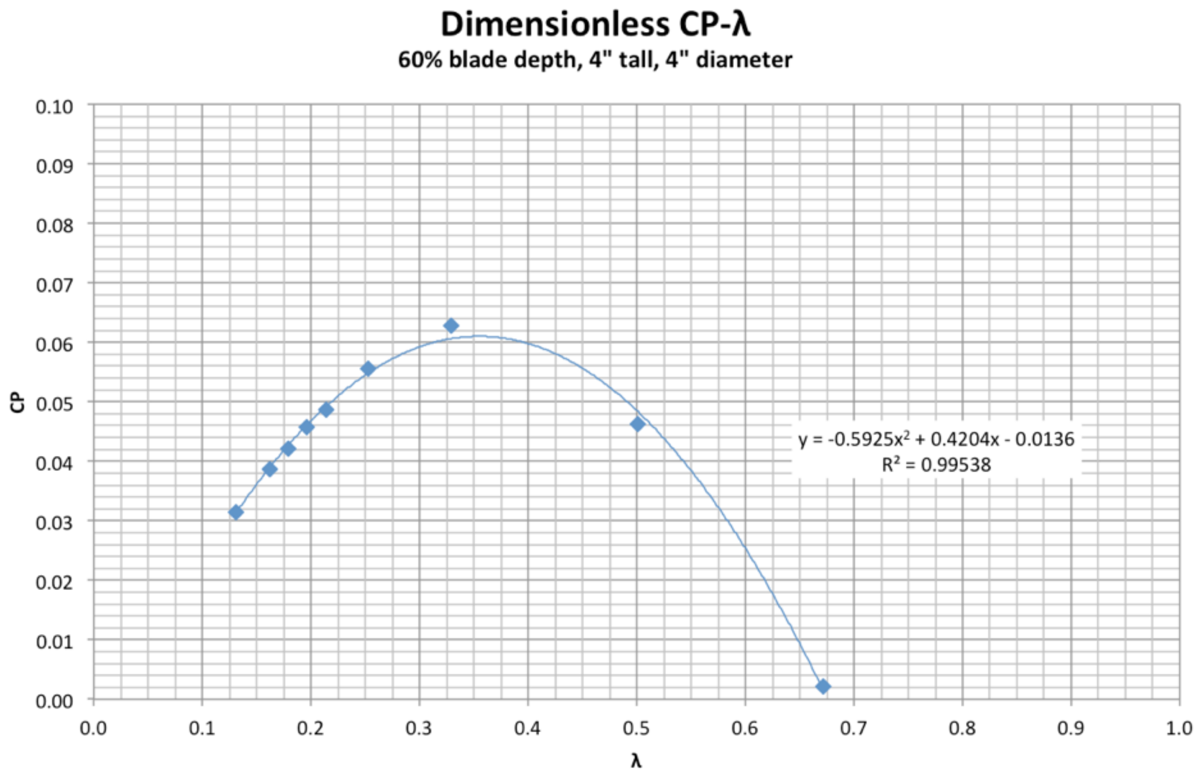


FIGURE 9

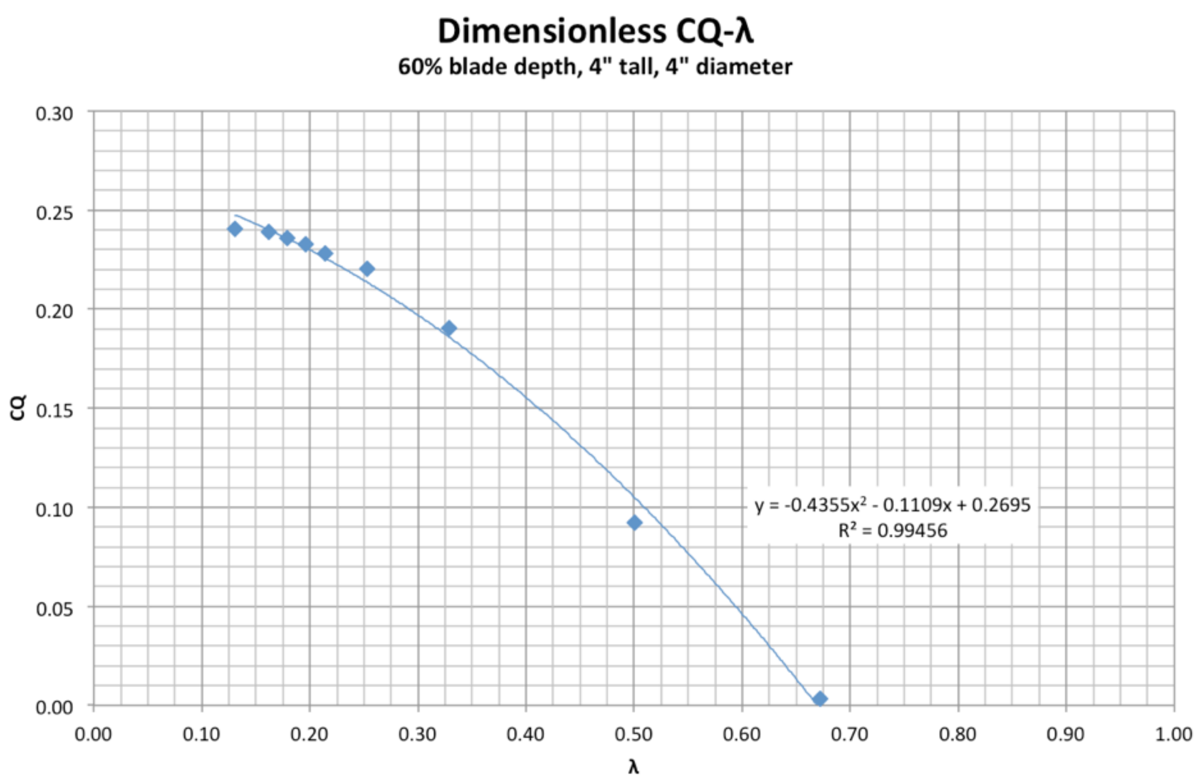
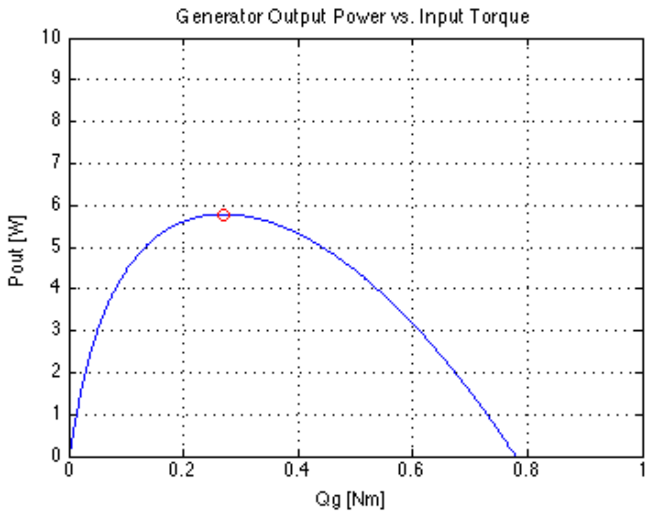


FIGURE 10

The scaled mechanical power output for the full size turbine is 8.38W (Q_t = 3.55 Nm, Ω_t = 2.41 rad/s), estimating a velocity of 5m/s. Inputting these parameters into the MATLAB script the team wrote to solve the generator problem yields the following result:




```
Power Output:   5.786786 [W]
Re:             34.733686 [ohms]
Gear Ratio:     13.086531
Qg:             0.271271 [Nm]
Wg:             34.733686 [rad/s]
I:              0.408172 [A]
```

FIGURE 11

Conclusion

FIGURE 12

The team constructed the full-scale turbine primarily from balsa wood, cardboard, and plastic sheeting, using the typical rib-and-spar technique (A.II Fig. 1, 2) found on many remote control planes, old-fashioned windmills, and aircraft. The turbines endplates were constructed of 0.25” foam core and 0.25” MDF (A.II Fig. 3). The goal of the design was to keep the turbine as light and inexpensive as possible. The full-scale turbine is pictured at left. See Appendix II for additional photographs.

Gear geometry was generated using the *Gear Template Generator*.[9] The transmission was two stages of 3.6:1 reduction (54:15 tooth ratio), resulting in a 12.96:1 gear ratio. Gears were laser cut from 0.25” MDF.

The electrical output power of the wind turbine during the demo test can be seen in Figure 13. The cyclical power output effects of the single stage vertical blade configuration is clearly visible. On the demo day, the team recorded an output power of 2.53W. This is lower than the predicted electrical output power. The team believes that the dominant sources of prediction error is an overestimate of wind speed. The team found the cart to be harder to push than expected, making the speed estimate of 5m/s too high. For a windspeed of 4m/s, the estimated mechanical turbine output is 4.29W, with maximum electrical power output is 2.78W. The incorrect estimate of wind speed is a combination of not having performed a test run with the cart and turbine, and team member fatigue.

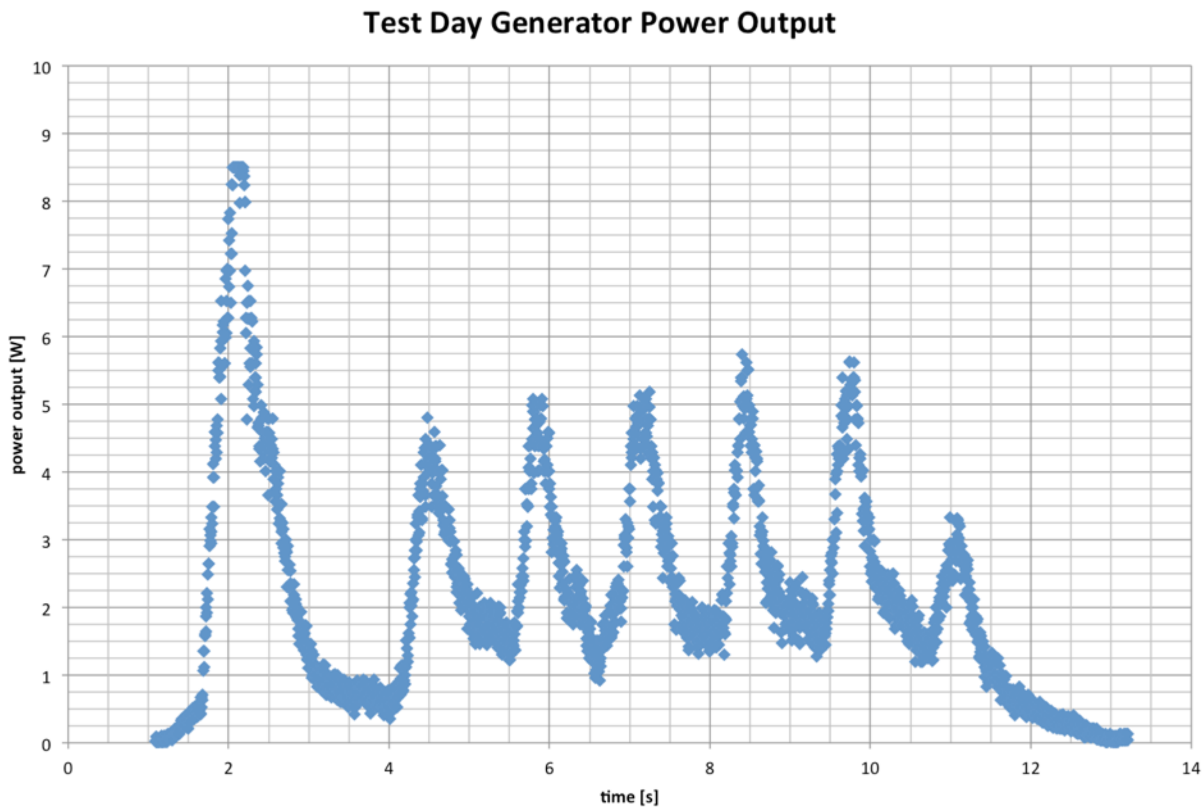


FIGURE 12

Were the team to construct a second VAWT, the following changes to design and design process would be implemented to improve overall performance:

- scaled testing of turbines would further investigate cup depth ratio effects on C_P
- wind tunnel tests for characterizing full-size turbine would reflect actual aspect ratio
- multi-stage turbine with offset rotors
- drag shield/nozzle
- more extensive testing of full-scale conditions for better wind speed estimates

[1] Letcher, T. (2010). Small Scale Wind Turbines Optimized for Low Wind Speeds. Retrieved from http://hdl.handle.net/1811/45531

[2] Letcher. Small Scale Wind Turbines.

[3] Travis J. Carrigan, Brian H. Dennis, Zhen X. Han, and Bo P. Wang, “Aerodynamic Shape Optimization of a Vertical-Axis Wind Turbine Using Differential Evolution,” ISRN Renewable Energy, vol. 2012.

[4] Modi, V.J., Fernando, M.S.U.K, & Roth, N.J. (1990). Aerodynamics of the Savonius rotor: experiments and analysis. Retrieved from http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=747953

[5] Letcher. Small Scale Wind Turbines.

[6] C-fec VAWT turbine design. http://www.c-fec.com/turbine/

[7] Fiene, J. (2014). *DC Brushed Motors* [PDF document]. Retrieved from MEAM.Design Wiki, https://alliance.seas.upenn.edu/~medesign/wiki/uploads/Courses/510-11C-L03.1.pdf

[8] Modi, Fernando, Roth. Aerodynamics of the Savonius Rotor.

[9] Wandel, M. *Gear Template Generator*. http://woodgears.ca/gear/index.html

APPENDIX I — Technical Figures

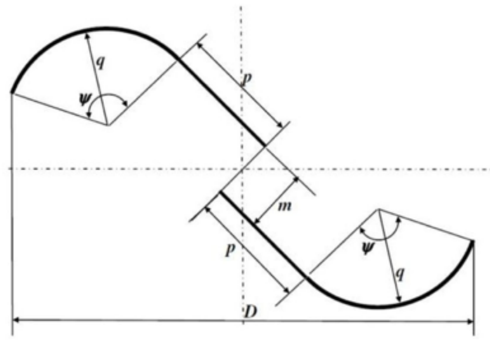


FIGURE 1

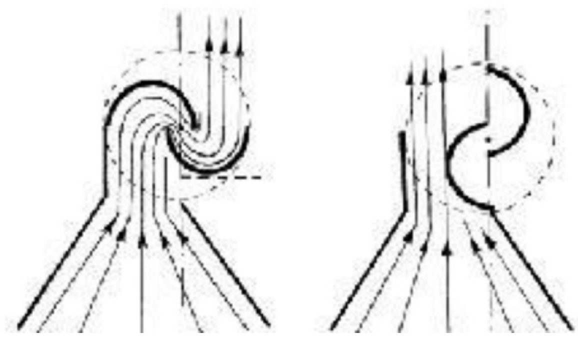


FIGURE 2

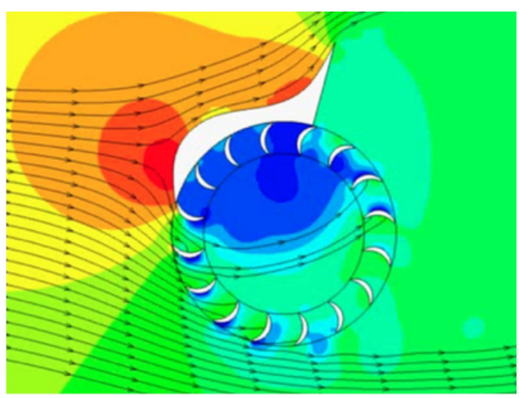


FIGURE 3

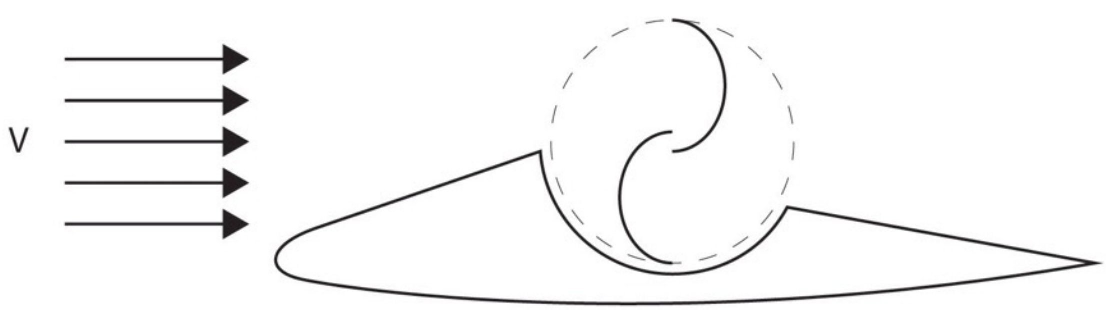


FIGURE 4

APPENDIX II — Full-Scale Turbine Photographs

FIGURE 1 — RIB AND SPAR METHOD

FIGURE 2 — RIB AND SPAR METHOD

FIGURE 3 — BOTTOM END PLATE

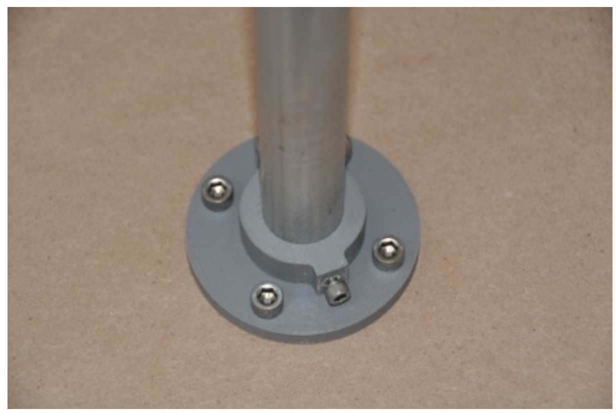


FIGURE 4 — TOP DRIVE COUPLER

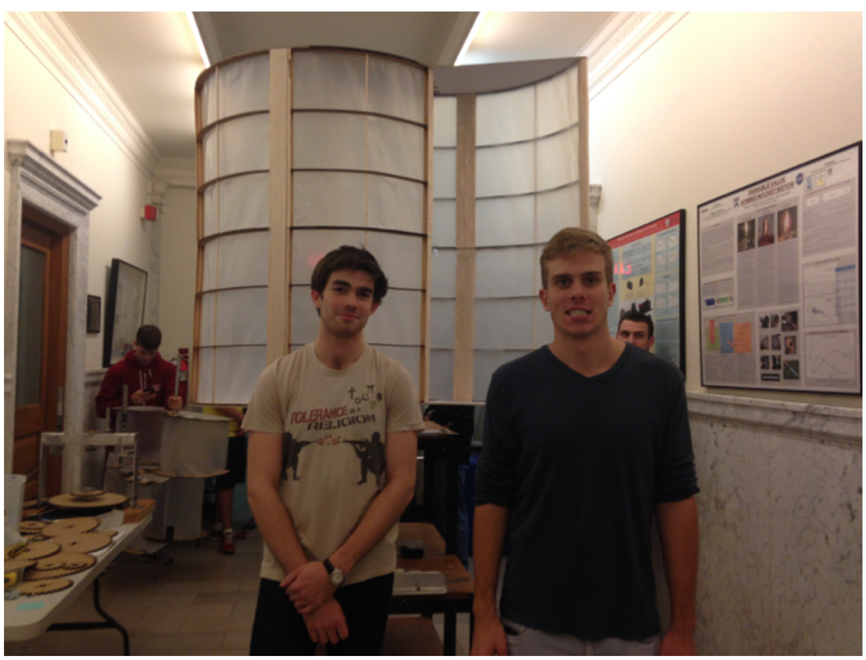


FIGURE 5 — THE VAWT TEAM

APPENDIX III — MATLAB code

```
%% Generator Optimization
% Takes in the measured/calcualted power generated by the turbine and
% determines the speed for which the generator is most efficient at that
% power.

clear all
close all
clc

%% Variable Initialization

% set motor properties
Qf = 0.0889;          % [Nm]      generator friction torque
Rg = 3.15;            % [ohms]    internal motor resistance
kt = 0.4468;          % [Nm/A]    torque constant
ng = 1.;              %           gearbox efficiency

% initialize variables
% Qg    generator torque
% Wg    generator spindle speed
% I     current
% Re    external resistance
% Pout  output power
syms Qg real;

% prompt for turbine output parameters
prompt = 'Input estimated turbine output power [W]:\n';
Pt = input(prompt);
prompt = 'Input estimated turbine output torque [Nm]:\n';
Qt = input(prompt);

%% Equation Initialization

% put variables in terms of Qg
I       = (Qg - Qf)/kt;
Wg      = (ng*Pt)/(Qf+Qg);
Re      = (Qg*Wg)/(I^2) - Rg;
Pout    = (((Qg-Qf)/kt)^2)*(((Qg*ng*Pt)/(Qf+Qg))/(I^2) - Rg);
G_ratio = Qt/Qg;

%% Pout Maximization

% set steps to evaluate
Qg_steps = linspace(0,1,1000);

% calculate Pout for each step
Pout_steps = double(subs(Pout, Qg, Qg_steps));

% find maximum Pout and location of maximum Pout
[Pout_max, index] = max(Pout_steps);
```

```
% find corresponding Qg
Qg_ideal = Qg_steps(index);

% plot Pout vs Qg, and highlight maximum
plot(Qg_steps, Pout_steps)
axis([0 1 0 10])
hold on
plot(Qg_ideal, Pout_max, 'ro')

% format plot nicely
title('Generator Output Power vs. Input Torque')
xlabel('Qg [Nm]')
ylabel('Pout [W]')
grid on

%% Calculate Corresponding I, Wg, Re, G

I_ideal = double(subs(I, Qg, Qg_ideal));
Wg_ideal = double(subs(Wg, Qg, Qg_ideal));
Re_ideal = double(subs(Re, Qg, Qg_ideal));
G_ideal = double(subs(G_ratio, Qg, Qg_ideal));

%% Print out results

fprintf('\n');
fprintf('Power Output:  %f [W]\n', Pout_max);
fprintf('Re:            %f [ohms]\n', Re_ideal);
fprintf('Gear Ratio:    %f\n', G_ideal);
fprintf('Qg:            %f [Nm]\n', Qg_ideal);
fprintf('Wg:            %f [rad/s]\n', Re_ideal);
fprintf('I:             %f [A]\n', I_ideal);
```